A Tour of the Calculus, *D. Berlinski*, Random House of Canada, Ltd., 1996, pp: 314, ISBN 0679426450 (hc); Price: CA$39.

Had it not been for the development of calculus, science would not have advanced any further than perhaps the invention of the wheel. Although everybody passes calculus courses, only a handful get enough insight to analyze a real-life phenomenon using the principles of calculus.

David Berlinski takes us through the very foundations of calculus in this unconventional and novel book. *A Tour of the Calculus* is really a fantastic, discerning, and entertaining introduction to calculus that explicates the relationship between calculus and real-world applications in a remarkably interesting manner.

The book can be roughly divided into two halves. The first half deals with the fundamental concepts that underpin calculus, while the second half is the real calculus in which the author describes the relationship of differentiation with rate of change, and the relationship of integration with area under curves.

The first half comprises the first fifteen chapters of the book in which Dr. Berlinski provides an informal history of the development of the subject. Calculus is gradually unveiled in a historical context (apparently fictitious) as Newton, Leibnitz and others discovered it.

The author starts from the very foundations, such as symbols, numbers, and Cartesian coordinates along with the Pythagorean theorem. He introduces the basics of numbers systems in the following chapters by proving that "$\sqrt{2}$ is irrational". Then in Chapters 7 and 8 he treats Dedekind cuts in reasonable depth, and follows this with a formal introduction to the real-number system and its completeness. Chapters 10 and 11 deal with functions and their general categories. Here I wish the author had spent a little more time in explaining slopes and curvature of functions.

Towards the end of what I had called the first half, Dr. Berlinski introduces the idea of speed; the concept that leads to the development of calculus. By introducing instantaneous speed, Dr. Berlinski marvelously introduces the concept of limits without explicitly mentioning them. Chapter 14 deals with sequences and series. However, I would have preferred it if the book had presented its thorough treatment of sequences and series only after formally introducing the idea of limits. There is virtually no mention of the convergence of series, although the limits of sequences are described in an appendix to the chapter. In the last chapter the author spends a great deal of time (very appropriately) explaining the concept of limits and the significance of continuity.

The second half (Chapter 16 onward) deals with differential and integral calculus. Having laid the necessary foundations of real-number system, limits and continuity, Dr. Berlinski goes on to explain the concepts of differentiation, the mean-value theorem, the area under a curve, integration and the fundamental theorem of calculus.

Chapter 16 gives an introduction to derivatives using the concept of limits. In the next chapter, the concept of differentiation is further elaborated with geometrical arguments. The same chapter gives rules of differentiation of the special functions presented in Chapter 11. In the following chapter, the notions of maxima and minima are treated with somewhat less rigor than they deserve. Chapter 19 deals with the mean-value theorem in great detail; its proof is given in an attached appendix. In Chapter 20 the author provides a brief introduction to antiderivatives with the help of some examples. The next four chapters deal with the area under a curve and Riemann integration. The fundamental theorem of calculus is treated in great detail in Chapter 25.

The important thing about Dr. Berlinski is that he presents one concept at a time, which enables the reader to grasp it and link it with the previous concepts. Throughout the book, he never lets mathematical symbolism overpower his words; in fact, he makes very little use of symbols to describe the concepts. Most of the functions are given as functions of time, the usual case in many real-world applications. And since Dr. Berlinski begins calculus with an introduction to the concept of speed, this choice makes sense.

Although the concepts in the book are mainly introductory, sometimes the author goes surprisingly deep into concepts that even those students who are well-versed in mathematics would find difficulty grasping (e.g., completeness of real numbers, the mean-value theorem, the fundamental theorem of calculus). But at other times he only skims through very important concepts (e.g., differentiation and integration in general, and maxima and minima). This variation in depth shows that Dr. Berlinski emphasizes the philosophical and theoretical aspects of calculus rather than the computational and applied aspects.

Some of the material that is rarely found in other calculus books are: a detailed treatment of number systems that invokes the Dedekind cut and the completeness of real numbers (topics usually treated in books on analysis); and the author's classroom experiences and historical notes (though mainly fictitious) featuring Newton, Leibniz, Galileo, Cauchy, Euler, Gauss, Riemann and others.

Dr. Berlinski presents complicated concepts in the style of prose, which is quite a rare approach in mathematical texts. The narrative and non-mathematical style makes the book more accessible to a casual reader; but people with some background in calculus tend to get bored with such non-mathematical language. It is a commendable attempt by Dr. Berlinski to renew the blasé students' interest through this approach.

He explains almost all the concepts that introductory calculus books generally include, but not in the usual dry manner as to define, to state, to prove, to infer, and to ask questions. Dr. Berlinski's approach is to first provide motivation for the concept by narrating some historical event, or a fictitious conversation about some deficiency in the theory as presented so far, and then to build on it intuitively. This way the reader, even before starting the next chapter, gets a good idea of what the next concept might be. It is obvious to those with some calculus background that Dr. Berlinski was able to hide the difficulty of some of the very complex concepts with his words and imagery. Unfortunately, some parts of this stimulating and vivid book digress too far away from the subject—at times one feels over-burdened with the imagery and the metaphorical language.

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In summary, the book is not about applying calculus but about calculus itself. The book has very few practical examples and no practice exercises. After reading this book, a novice reader might not learn an awful lot about solving problems with calculus but will surely be able to appreciate the elegance of the subject. The mathematically sophisticated reader will also find this book a valuable resource for acquiring the deeper meaning behind various concepts of calculus. If one is more inclined towards “algorithmic calculus”, then he or she probably would not like this book.

A Tour of the Calculus can in no way be recommended as a textbook for a calculus course, but it can be very good supplementary reading. I recommend this book to every college teacher of calculus: Dr. Berlinski’s unique perspective of calculus will be really helpful for those teaching introductory calculus courses.

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