

MICROTRONS IN CANADA

by Paul A. Redhead

The proposal for the microtron, a uniform field cyclic electron accelerator, was first published in 1944 by Veksler in the USSR. The world's first microtron was constructed and operated at the National Research Council of Canada in 1947 producing electrons of 4.6 Mev energy. The racetrack microtron, using a sectored magnet, was proposed independently by Moroz and Roberts in 1958 and the world's first racetrack microtron was constructed at the University of Western Ontario in 1961 producing 5 Mev electrons. The history of these early microtrons and their descendants is briefly outlined.

INTRODUCTION

The first cyclic accelerator, the cyclotron, was operated^[1] by Lawrence in 1931 and by 1932 protons of 1 Mev had been produced. By the end of the decade it was evident that proton energies above about 15 Mev in a cyclotron were not possible because of the relativistic change in mass. In 1941 Kerst^[2] built the first betatron which used a varying magnetic field to induce an electric field which accelerated a beam of electrons moving in a circle. By 1950 electron energies of about 300 Mev were reached with betatrons, the maximum energy being limited by synchrotron radiation from the orbiting electrons.

A new acceleration mechanism which was based on the idea of phase stability was first proposed by Oliphant in 1943. Oliphant was prevented from publishing his idea by wartime censorship, the concept of phase stability was later published independently by Veksler^[3] in Russia (1944) and McMillan^[4] in the USA (1945); all three noted that charged particles which cross the accelerating gap at a phase of the r.f. field close to the optimum phase would oscillate in phase and remain stable. Acceleration of the particles could then be achieved by varying the frequency of the r.f. field or by increasing the magnetic field. This concept led to the development of the synchrotron in which the frequency was scanned.

In the paper by Veksler in 1944 it was proposed that electrons could also be accelerated to relativistic energies in a

uniform magnetic field by multiple passages through a microwave cavity, the energy gained by the electron on each passage having a simple relationship to the rest energy of the electron. Veksler pointed out that when a relativistic particle is periodically accelerated in a magnetic field the difference in time to perform successive orbits is a constant, independent of the total energy of the particle. The concept of the microtron was also mentioned in lectures in the USA, but not published, by L.W. Alvarez as early as 1939 and by J.S. Schwinger in 1945. Schiff^[5] in 1946 discussed Veksler's proposal and coined the name microtron. The microtron was independently proposed by Itoh and Kobayashi in Japan at a

meeting of the Physical Society of Japan in 1947 and published later in 1949^[6].

Figure 1 is a schematic diagram of a conventional microtron with a uniform magnetic field showing the circular electron orbits of increasing diameter that are tangent to one another as they pass through the accelerating gap of a microwave cavity. Since the electrons rapidly approach the velocity of light they gain energy and momentum, but almost zero velocity, in passing through the cavity. One advantage of the microtron is that the energy gained by the electrons in one revolution is so large (unlike the betatron) that the energy loss by synchrotron radiation is insignificant. Another advantage is that the electrons in the final orbit are easily ejected. Phase stability in the microtron results in stable oscillations in the r.f. phase at which the electron crosses the accelerating gap. An electron crossing the gap after the equilibrium phase gains less than the resonant energy and the next orbit takes less time so that the electron comes to the next gap crossing nearer to the resonant phase.

The early development of microtrons has been reviewed by Kaiser^[7] in 1956, by Paulin^[8] in 1962, in a book by

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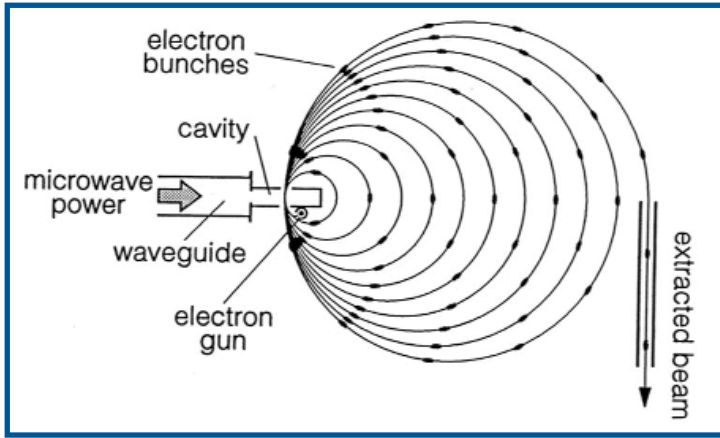


Fig. 1 Schematic diagram of the electron orbits in a uni-form-field microtron.

Kapitza and Melekhin^[9] in 1969, by Rosander^[10] in 1980, and Scharf^[11] in 1986.

THEORY OF THE UNIFORM-FIELD MICROTRON

The condition for resonant acceleration is that the orbital period increases by an integral number of r.f. periods from one orbit to the next. A simple analysis of the resonance conditions follows.

The orbital period of a charged particle in a uniform magnetic field B is

$$T = \frac{2\pi m}{eB} = \frac{2\pi W}{eBc^2}, \quad (1)$$

where e is its charge, m its relativistic mass, and the total energy of the particle is $W = mc^2$. We see that the orbital period of the particle is directly proportional to its total energy. If the energy gain of the particle at each crossing of the gap is ΔW , then the particle energy in the n^{th} orbit is

$$W_n = W_0 + W_i + n\Delta W, \quad (2)$$

where $W_0 = m_0c^2$ is the rest energy of the particle and W_i is its injection energy. Then the orbital period in the n^{th} orbit (neglecting transit time in the cavity) is

$$T_n = \frac{2\pi}{eBc^2} (W_0 + W_i + n\Delta W). \quad (3)$$

Then the difference in time to perform successive orbits is

$$\Delta T = T_{n+1} - T_n = \frac{2\pi\Delta W}{eBc^2}, \quad (4)$$

note that ΔT is independent of the particle energy and with constant B depends only on the energy gained per revolution. To obtain resonant acceleration it is thus necessary to adjust ΔW and B so that

$$\frac{2\pi\Delta W}{eBc^2} = b\tau \quad (5)$$

where b is any integer except zero and τ is the period of the r.f. field.

The second resonant condition concerns the orbital period of the first revolution which is given by

$$T_1 = \frac{2\pi(W_0 + W_i + \Delta W)}{eBc^2} = a\tau \quad (6)$$

where a is any integer except zero or unity.

The fundamental mode of operation is when W_i is zero and $\Delta W = W_0$ then $a = 2b$. This resonance condition can be achieved with electrons starting from rest if the energy gained at each gap crossing is made equal to the rest energy of the particle $\Delta W = W_0$, Eq. 6 then reduces to

$$\frac{2\pi(W_0 + \Delta W)}{eBc^2} = a\tau. \quad (7)$$

Combining equations 5 and 7 the resonant magnetic field and accelerating voltage V are

$$B = \frac{2\pi m_0 c}{e(a-b)\tau} = \frac{2\pi W_0}{ec(a-b)\lambda} \quad (8)$$

where the r.f. wavelength $\lambda = \tau c$, and

$$V = \frac{\Delta W}{e} = \frac{W_0}{e} \frac{b}{(a-b)}. \quad (8a)$$

Taking the simplest condition with $a = 2$ and $b = 1$ and a convenient microwave frequency of 2.8 GHz the resonant conditions for electrons are $B = 0.0995$ T and $V = 511$ kV.

The major problem in building a microtron is to design an r.f. system which produces a peak voltage of about 0.5 MV across the accelerating gap of a microwave cavity, this requires a cavity Q of the order of 10^4 . With electric fields of this magnitude there is no problem in producing adequate electron current by field emission from the cavity walls.

The phase focusing process in the microtron and the synchrotron are essentially similar. Charged particles which enter the accelerating gap after the resonant phase will be phase focused back towards the resonant phase on later gap crossings. Veksler showed that electrons crossing the gap after the peak of the r.f. wave in the microtron would have inherent phase stability. The phase stability of the microtron has been treated in detail by Henderson^[12] et

al. It was found in an early microtron operating at 2.8 GHz with $a = 2$ and $b = 1$ that resonant acceleration could be obtained from 0.09 to 0.12 T. It has been shown^[13] that the region of phase stability in a conventional microtron is

$$\begin{aligned} &90 \text{ to } 122.5^\circ \text{ for } b = 1 \\ &90 \text{ to } 107.7^\circ \text{ for } b = 2 \\ &90 \text{ to } 102.0^\circ \text{ for } b = 3 \end{aligned}$$

The $a = 2$ and $b = 1$ mode, the fundamental mode, is to be preferred for its large phase stable region as well as having the largest energy gain per revolution and the largest magnetic field.

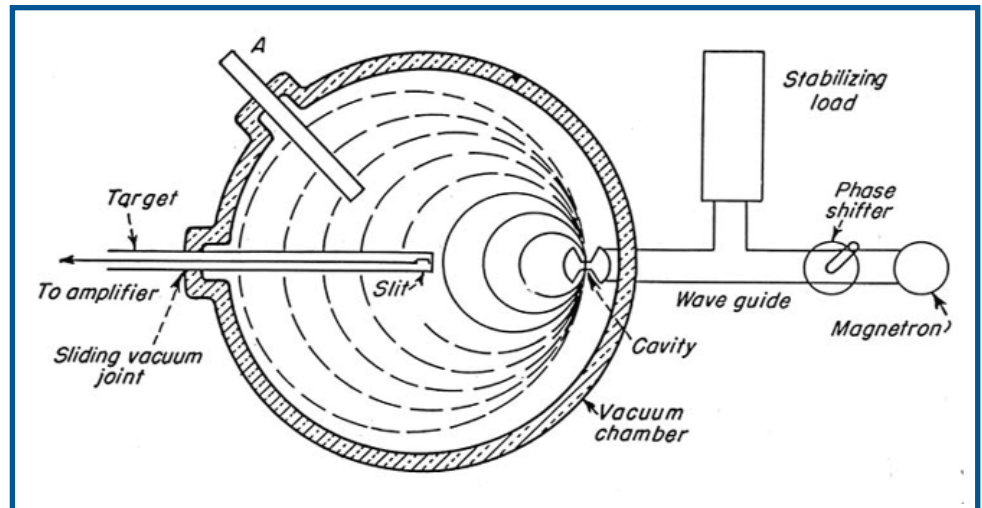


Fig. 2 Simplified diagram of the first microtron.

THE FIRST OPERATING MICROTRON

During the second world war the major centre of research and development on radar in Canada was the Radio Branch of the Physics Division at the National Research Council. When the wartime task of the Radio Branch ended in 1945 the Branch had a considerable body of expertise in microwave systems together with a large amount of surplus microwave equipment. W.J.Henderson had served in the Radio Branch during the war; in 1945 he read the English translation of Veksler's paper and realized that the Radio Branch was in an excellent position to construct a prototype model of the microtron (or electron cyclotron as it was then called) to test Veksler's theory.

Design of the microtron started in 1946, it was decided to operate in the fundamental mode with $a = 2$ and $b = 1$ at a frequency of 2.8 GHz where a tunable magnetron (RK-5586) with a pulsed power output of 300 kW was available. The maximum available magnetic field was about 0.13 T. It was decided to rely on field emission from the cavity walls in the r.f. field (10^6 V/cm at the corners of the cavity) to produce the initial electrons, hence $V_i = 0$. Figure 2 shows a simplified diagram of the microtron with the 8 orbits possible in the vacuum chamber of 36 cm diameter, a moveable target allowed measurement of the current in the orbits. Figure 3 is a photograph of the NRC microtron in about 1948.

The major microwave problem was to design and build a microwave cavity of a shape that would have a short accelerating gap and not obstruct the small radius electron orbits, provide adequate field emission and yet not cause sparking, and have a large shunt resistance or Q so as to provide a peak r.f. voltage at the accelerating gap of over 0.5 MV. These requirements were not easy to achieve simultaneously. The cavity shape chosen was an ellipsoid-hyperboloid with a 13.8 mm gap (several different designs were tested, one successful design is shown in Figure 4). Hugh LeCaine designed a special instrument

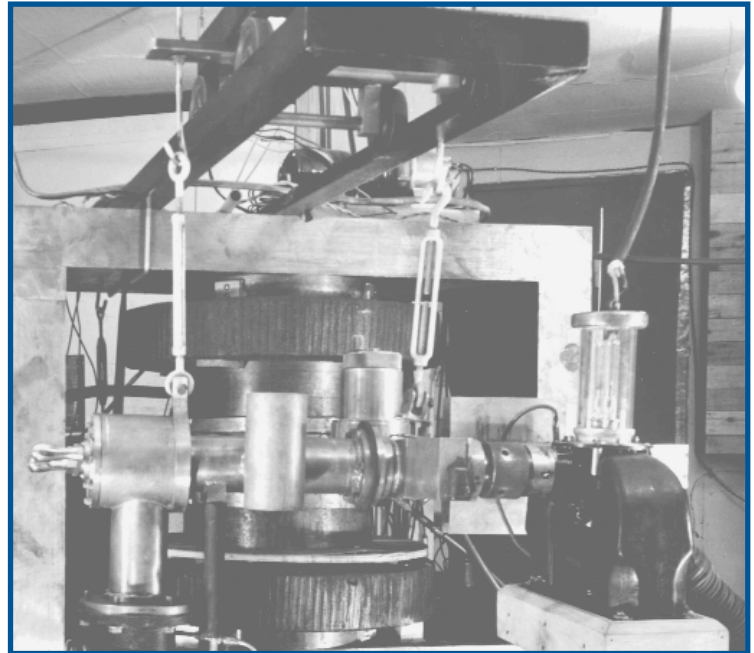


Fig. 3 Photograph of the NRC microtron in 1948. The tunable magnetron is visible in the right foreground; in the left foreground is the vacuum system with a cold-trap, vacuum valve, and part of the diffusion pump showing. The phase shifter can be seen in the centre.

for measuring the high values of cavity Q. The Q values of copper cavities were about 10,000 with estimated shunt resistances of 2×10^6 ohms which were 80% of theoretical values^[14].

The microtron first operated in the summer of 1947 producing electrons of 4.6 Mev in the eighth orbit^[15]. Legend has it that after the microtron first operated, one night at the NRC Field Station outside Ottawa, it was decided to celebrate the achievement in the beer parlours of Ottawa (bars serving liquor were still outlawed in Ontario). At about midnight it was decided that it would be remiss not to inform Veksler that his idea worked. A visit was

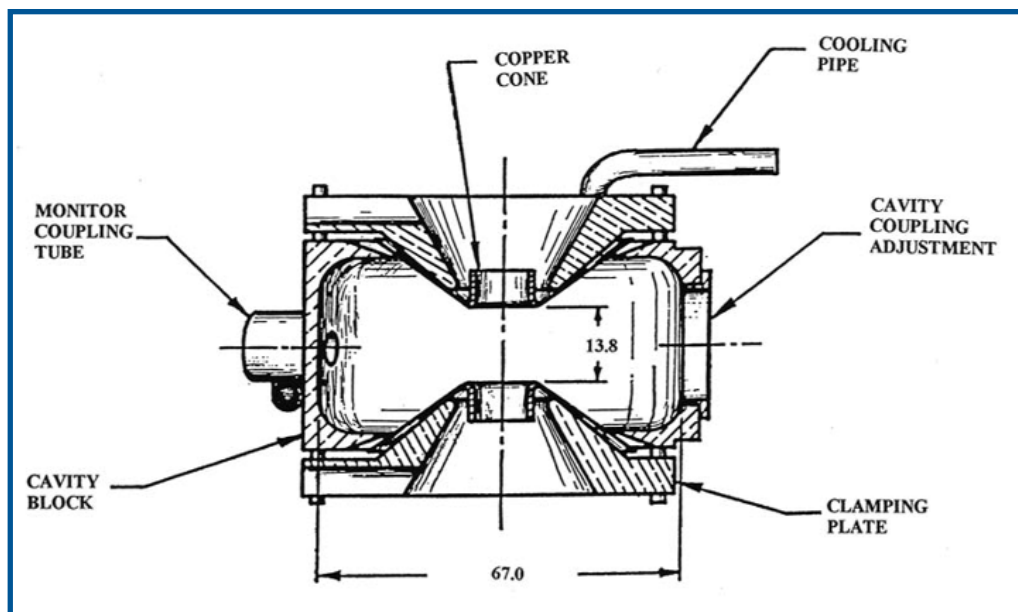


Fig. 4 One of the cavity designs used in the NRC microtron. All dimensions in mm.

then paid to the telegraph office on Sparks street which had been shut for some hours. Loud and erratic banging brought the telegrapher, who lived over the office, to the door and he was persuaded to send a telegram to Veksler care of the Academy of Sciences in Moscow. No reply was expected as Russian-Canadian relations had been somewhat distraught since the Gouzenko affair only two years before. Some two or three years later a message of congratulations from Veksler was received by hand of a Canadian scientist who had attended a conference in Europe where Russian scientists were present.

Figure 5 shows the electron currents in the eight orbits where it can be seen that the loss of electrons after the second orbit is small until the eighth orbit where the magnetic field has decreased significantly.

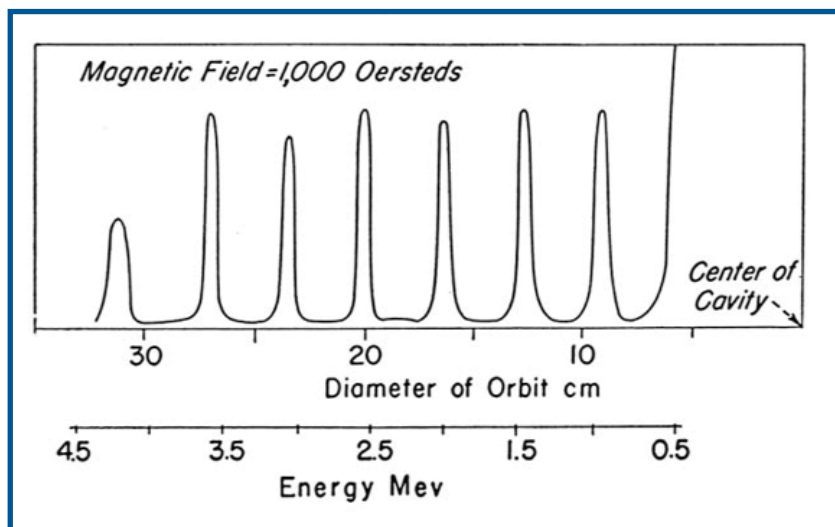


Fig. 5 Electron currents in the eight orbits of the NRC microtron.

In 1947 and 1948 the performance of the microtron was improved^[16] and attempts made to inject electrons into the first orbit from a thermionic cathode without much success. The maximum electron current observed in the 7th orbit was 3 mA.

An academic who had been appointed to a senior position in the scientific bureaucracy of Ottawa asked to see the microtron in operation in 1948. He appeared quite disappointed to see the evidence of resonant acceleration of electrons since the theory of the microtron assumed the validity of Einstein's theory of special relativity to which he did not subscribe. He departed in some irritation, frustrated by his inability to explain the operation of the microtron from classical theory.

In 1951 the microtron was transferred to the Physics Department at the University of Western Ontario where it was modified^[17] and used for some years to study the generation of microwaves at millimeter wavelengths^[18].

In the 1950s and early 1960s some 15 uniform-field microtrons were built around the world^[19,20], with maximum energies ranging from 2 to 29 MeV. The first microtron in the USA was built at the Naval Research Laboratory^[21] in 1952 and operated at 9.4 GHz. The largest machine was constructed at University College of the University of London^[22] and first operated in 1958 having 56 orbits and a final energy of 29 MeV. Most of these microtrons used field emission from the cavity walls as the electron source.

The microtron was relatively simple to build and operate, the synchrotron radiation loss was negligible, and the beam was easy to extract; however, the electron current available was modest and was being overtaken by the linear accelerator. A considerable effort was begun in the USSR under Kapitza to construct improved microtrons, the first Russian microtron^[6] appeared in 1958 having 12 orbits and a final energy of 6 to 7 MeV; electrons were injected from a thermionic emitter. Considerable effort was put into devising systems for efficiently injecting electrons into the first orbit resulting in increased electron currents in the final orbit.

The major use of the microtron has been as an injector into synchrotrons for which it is particularly suited. The microtron was first used as an injector into the synchrotron at Lund in Sweden^[23] in 1964, followed by an injector at Frascati in 1970 and two microtron injectors in the USSR at the Lebedev

Institute and the Tomsk Polytechnical Institute. The first microtron in the USA designed as an injector was built for the synchrotron radiation source at the University of Wisconsin-Madison in the early 1970s.

THE FIRST RACETRACK MICROTRON

J.S.Schwinger suggested in 1945 a modified form of the microtron with a split magnet and a series of resonant cavities in the field-free gap, this was later called the race track microtron. Detailed proposals for the design of race-track microtrons were first made by Moroz^[24] in 1956 and Roberts in 1958^[25]. The main advantages of the race-track design are that it is much simpler to inject electrons in the field free region around the accelerating gap(s) and that focusing can be achieved at each crossing of the magnet gaps^[26] to ensure axial stability.

The first racetrack microtron was operated at the University of Western Ontario^[27] in 1960 (Figure 6 shows a schematic diagram and Figure 7 is a photograph taken in 1964) it had four magnetic sectors, after a design first proposed by Roberts^[24], the pole pieces could be moved during operation to permit varying the drift space length; the fringe fields provided beam focusing. The magnetic field within the four sectors was uniform with a magnet gap of only 7.2 mm so that a field of 3 T over the 40 cm diameter could be provided with 100 W of power. The electrons were obtained by field emission from the cavity walls. The microtron was designed for 8 orbits but initially only 7 orbits could be used giving a final energy of 5 MeV. The resonance condition is unchanged from the conventional microtron and the orbit time for the n^{th} orbit is

$$T_n = \frac{2\pi}{eBc^2} (W_0 + W_i + n\Delta W) + \frac{l_n}{v_n} = (f + gn)\tau \quad (9)$$

where l_n is the drift space length for the n^{th} orbit, v_n the velocity in the n^{th} orbit, and f and g are integers. This condition can be satisfied by choosing

$$l_n = \left(f - g \left[\frac{W_0 + W_i}{\Delta W} \right] \right) \tau v_n \quad (10)$$

and

$$B = \frac{2\pi}{ec^2} \frac{\Delta W}{g\tau}. \quad (11)$$

The simplest case is when $v_n \approx c$ and the drift space length is constant, then

$$l = \left(f - g \left[\frac{W_0 + W_i}{\Delta W} \right] \right) \tau \quad (12)$$

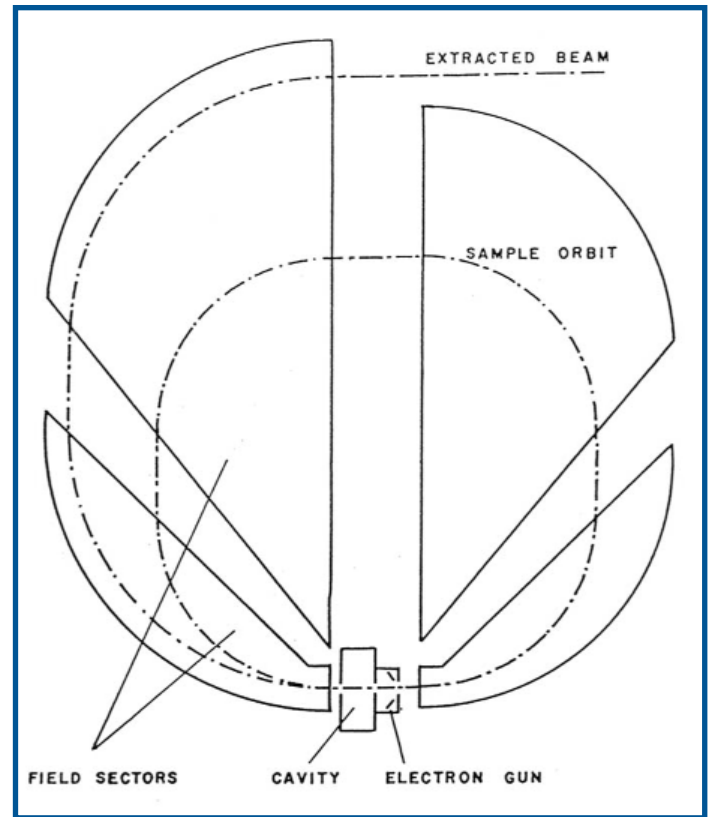


Fig. 6 Schematic diagram of the four-sector racetrack microtron first operated in 1960 at the University of Western Ontario.

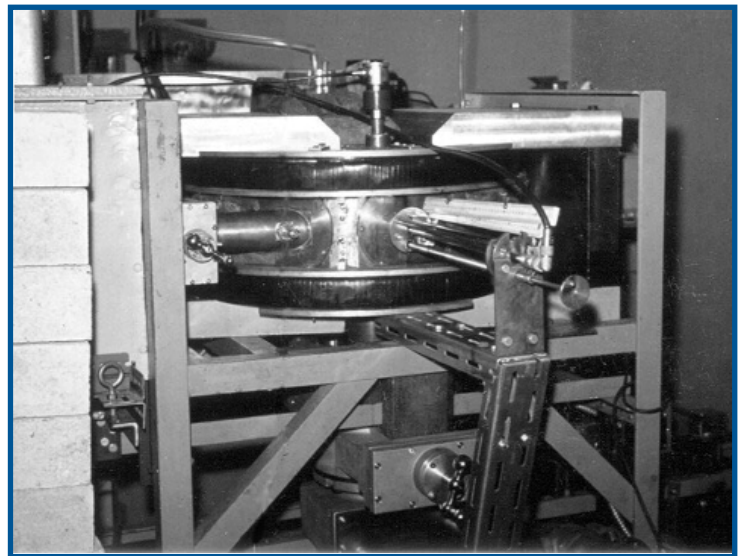


Fig. 7 Photograph of the first racetrack microtron at UWO in 1964.

The resonant condition can be satisfied for any energy gain ΔW by varying either the injection energy or the drift space length allowing continuously variable final energy.

An electron injection system using a lanthanum boride cathode was added and the modified microtron^[28] was

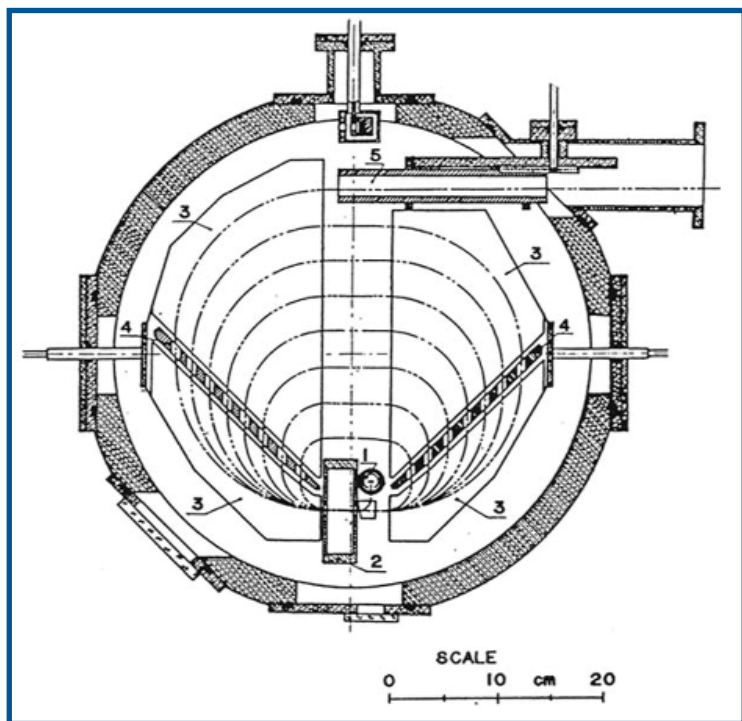


Fig. 8 Diagram of 8 orbit racetrack microtron built at UWO. 1) electron gun, 2) cavity, 3) pole pieces, 4) magnetic shields, 5) beam extractor

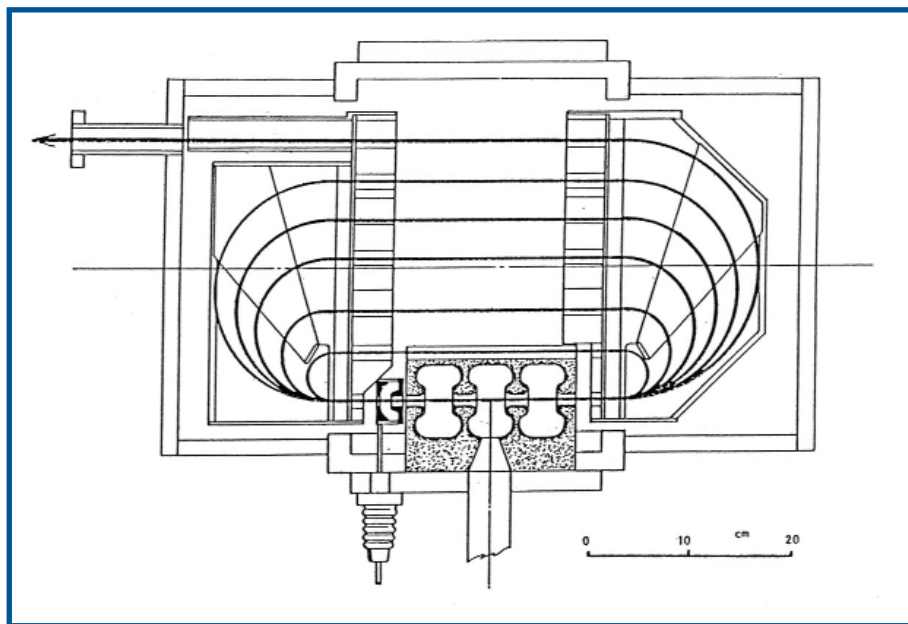


Fig. 9 Schematic diagram of 6 orbit, multicavity, racetrack microtron built at UWO.

capable of a maximum energy 6.3 MeV with a pulsed current of 40 mA. Figure 8 is a diagram of this microtron showing the magnetic poles moveable inside the vacuum chamber. A copy of this microtron was made in the USA and later transferred to India. A 2 MeV racetrack microtron was also built at UWO.

In the early 1970s the team at UWO developed a more ambitious form of the racetrack microtron^[29,30,31], using a

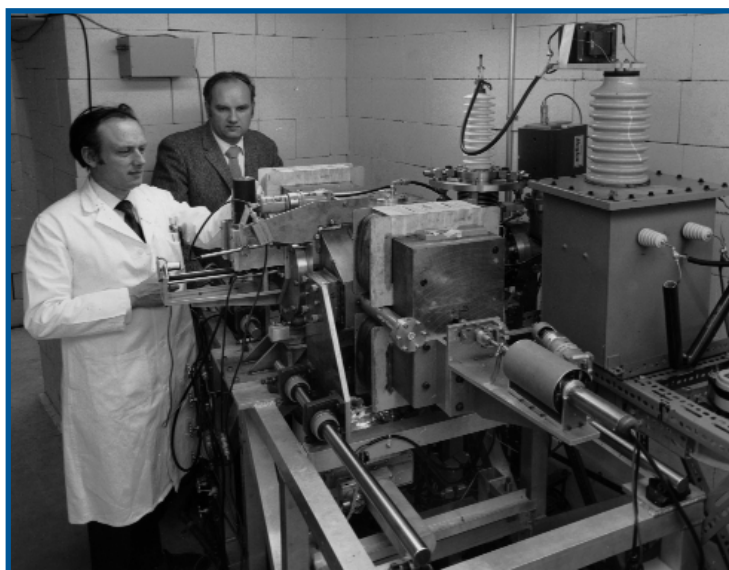


Fig. 10 Photograph of racetrack microtron shown schematically in Figure 9. Heinrich Fröelich stands behind the machine and Tim Thompson is on the left. The magnet structure could be unbolted from the vacuum chamber and moved out on the rails; in front of the magnet is a rotating coil fluxmeter; on the left is the mechanism to move a Faraday cup to measure orbit currents within the vacuum chamber.

very short linear accelerator section. The general form of this 6 orbit, microtron is shown in Figure 9. Figure 10 is a photograph of this machine. This microtron used a focusing system similar to that shown in Figure 8, but instead of two equally strong sector fields separated by a slanted field-free region, a magnet with three sectors, all with different field strengths (hills and valleys), was used on each side of the linear accelerator section. Electrons were injected from an annular filament surrounding the beam axis at energies up to 50 keV. The three cavities of the linear accelerator section gave a maximum energy gain ΔW of 3 MeV initially (later restricted to 2.5 MeV because of the deterioration of the cavity surfaces and increased sparking) yielding final energies variable from 9 to 15 MeV and electron currents of 30 mA with 6 orbits. Resonant operation of the microtron was achieved over a wide range of final energies by adjusting the magnetic field, the drift space length, and the energy gain per turn.

The drift space length was varied by moving the pole pieces. This microtron was developed with a view to its use in radiation therapy applications and was the largest one built with this kind of focusing.

In the early 1970s Atomic Energy of Canada entered into a contract with UWO to construct a race-track microtron capable of delivering an x-ray beam at a specified dose rate suitable for therapy purposes. The specified dose rate

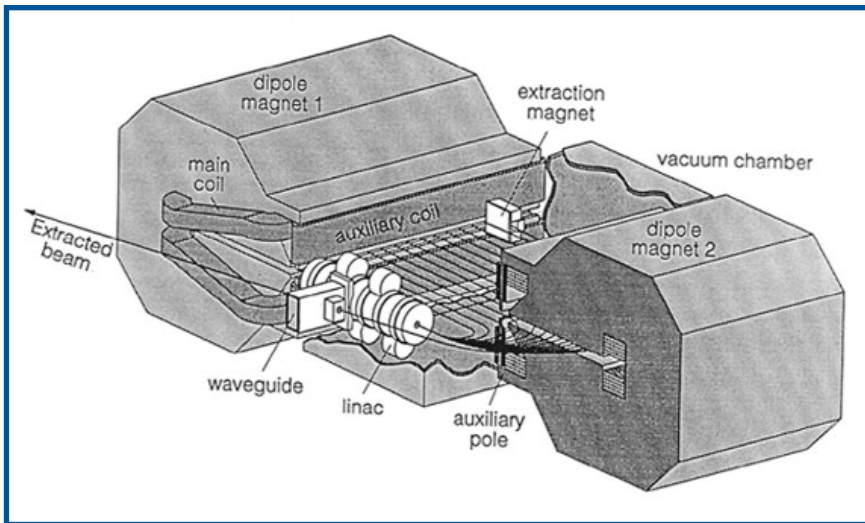


Fig. 11 View of a modern racetrack microtron. From *Electron Physics of Vacuum and Gaseous Devices*, by Miroslav Sedláček, 1996. Reprinted by permission of John Wiley & Sons, Inc.

In 1967 a new method of focusing to achieve axial stability was developed in the Royal Institute of Technology in Stockholm [32]. Two dipole magnets were used with a thin magnet with reversed polarity along the edge where the electrons entered, thus a narrow part of the fringe field region has a reversed field. This changes the axial defocusing into focusing and allows the use of only two dipole magnets. All racetrack microtrons since 1973 have used reversed-field focusing. Figure 11 is a cutaway view of a modern racetrack microtron [33] using reversed field focusing, the beam focusing in the dipole magnet of this type of microtron is explained in the same reference.

Further progress on microtrons proceeded after 1973 at several laboratories around the world as the use of microtrons as injectors into electron synchrotrons and storage rings developed. By 1987 the maximum energy achieved by a microtron had increased to 800 MeV, Figure 12 shows the maximum energy of various types of accelerators [34], including the microtron, as a function of time. All these later microtrons were descendants of the first microtron and first racetrack microtron developed in Canada

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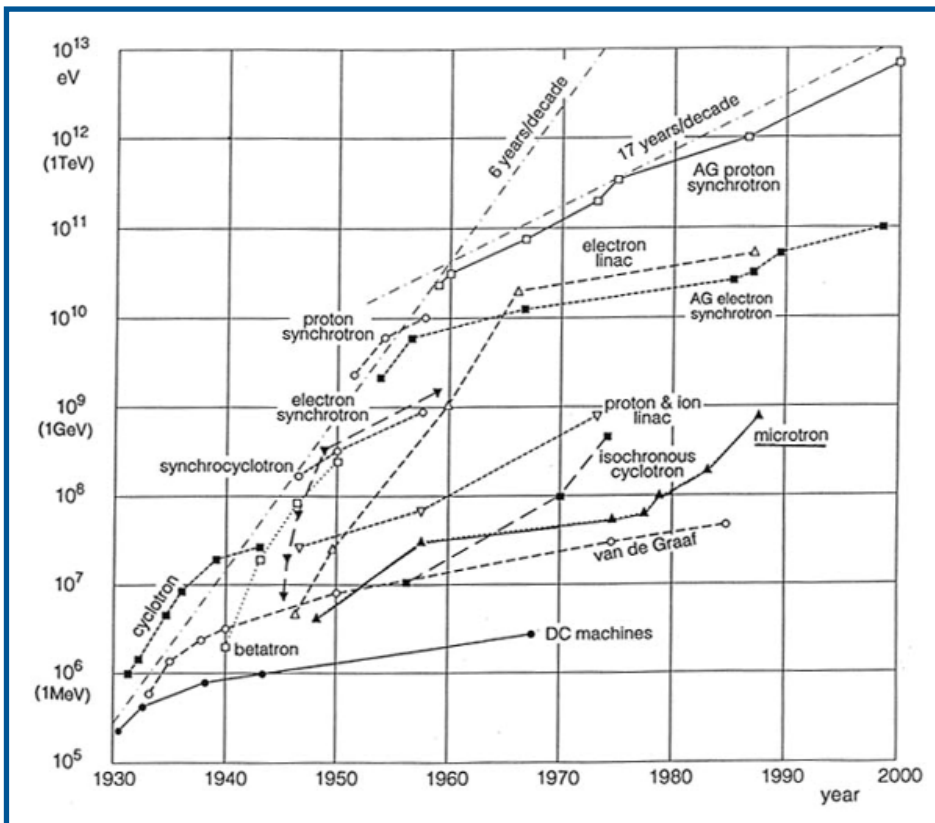


Fig. 12 Livingston diagram showing the increase in final energy of microtrons compared to other types of accelerators. From *Electron Physics of Vacuum and Gaseous Devices*, by Miroslav Sedláček, 1996. Reprinted by permission of John Wiley & Sons, Inc.

was achieved in 1974 and AECL decided to move the project to Ottawa for comparison with a linear accelerator alternative, the microtron program at UWO was then terminated Eventually AECL decided to proceed with the linear accelerator for their x-ray therapy unit.

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