# 2012 CAP Prize Examination

Compiled by the Department of Physics, University of Alberta

Tuesday, February 7, 2012 Duration: 3 hours.

Family name: ______________________ Given name: ______________________

Institution: ____________________________

**Instructions:**

1. You have 3 hours to complete this exam.
2. A scientific calculator is permitted but textbooks or reference materials are not.
3. There are 10 questions, each weighted equally. It is unlikely that you will be able to answer each question, therefore you should budget your time wisely.
4. Write your solutions on the pages provided. Use the back of the pages if more space is needed.
5. This exam consists of twenty two (22) pages.

**Physical constants and aide-mémoire:**

- elementary charge: $e = 1.6022 \times 10^{-19}$ C
- permittivity of free space: $\epsilon_0 = 8.842 \times 10^{-12}$ F/m $= 8.842 \times 10^{-12}$ C$^2$/(Nm$^2$)
- permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ H/m $= 4\pi \times 10^{-7}$ Tm/A
- speed of light: $c = 2.9979 \times 10^8$ m/s
- electron mass: $m_e = 9.1094 \times 10^{-31}$ kg
- proton mass: $m_p = 1.6726 \times 10^{-27}$ kg
- atomic mass unit: $m_{au} = 1.661 \times 10^{-27}$ kg
- Boltzmann constant: $k = 1.3807 \times 10^{-23}$ J/K
- Planck constant: $\hbar = h/(2\pi) = 1.0546 \times 10^{-34}$ Js
- Gravitational constant: $G = 6.6726 \times 10^{-11}$ Nm$^2$/kg$^2$
- Pauli matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

**Question 1**

A wind turbine consists of blades mounted on a horizontal axis approximately 50 m above the ground. In steady 10 m/s wind, the turbine produces a power of 600 kW. Estimate the power generated by the turbine when the wind speed goes up to 12 m/s. In all cases, assume steady wind. (*Hint*: Consider the amount of wind energy per unit time going through the area spanned by the blades.)

**Question 2**

A gram of carbon in the graphite form is to be cooled from 20°C to 100 mK. The specific heat of graphite closely follows the temperature dependence predicted by Debye in the form of $C_v = AT^3$. At 20°C, $C_v = 0.5$ J/(g°C). How much energy must be removed from a gram of carbon to cool it? How much heat will be dumped into the environment by an ideal refrigerator to cool the graphite?

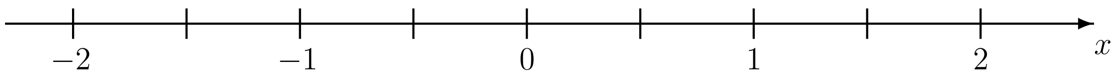
### Question 3

The so-called Cretaceous–Tertiary extinction event 65 million years ago is credited to an impact of an asteroid of about 10 km in diameter. Assuming that the density of an asteroid is similar to that of the Earth, 5000 kg/m$^3$, and that it is held together by gravity, calculate the energy needed to fully disintegrate it.

The largest tested hydrogen bomb was 50 MT, where MT denotes the megaton; a unit of energy equal to 4.184 $\times 10^{15}$ joules. Would such a device be enough to defend the Earth against the asteroid?

**Question 4**

In the space station orbiting the Earth, water can form a freely-floating ball (because of weightlessness). If we lightly touch this ball, its surface will start oscillating. The period of oscillations $T$ is proportional to what power of the radius $R$ of the sphere? (Find the exponent $x$ in $T \sim R^x$.) Describe your reasoning and mark your answer on the line below.



**Question 5**

A negative pion can be absorbed by a deuteron to produce two neutrons

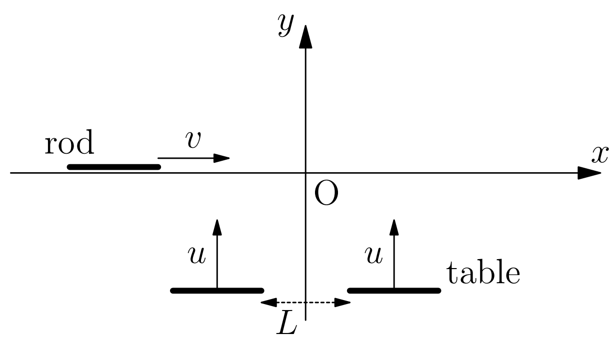
$$\pi + d \to n + n. \tag{1}$$

This reaction was the one used in the first experimental determination of the intrinsic parity of the pion. Use the fact that this reaction does take place (*i.e.*, is allowed) and the following information, to determine the intrinsic parity of the pion:

- The pion is absorbed from rest,
- The pion has spin 0,
- The deuteron has spin 1 and positive parity.

**Question 6**

A thin rod of rest length L moves longitudinally at relativistic speed $v$ in the $x$ direction as observed in an inertial frame $S$ (assume no gravity). A flat table whose surface is parallel to the $x$ axis, and is of negligible thickness, moves in the $y$ direction at relativistic speed $u$. The table has a rectangular hole, also of width L in the $x$ direction, such that the center of the rod will coincide with the centre of the hole at the origin point $O$.




Analyze the motion from the point of view of two observers: one in the $S$ frame mentioned above and another in the rest frame of the rod (call this frame $S'$), and provide the underlying arguments for an answer to the following question: Will the rod pass through the hole? Support your arguments by doing the required calculations.
[Source credit: R. Shaw, Am. J. Phys. 29, 365(1961)]

## Question 7

*Sundogs* are bright spots of light that can be seen beside the Sun on clear winter mornings due to the refraction of sunlight through thin hexagonal ice crystals that are typically aligned parallel to the ground. Taking the index of refraction of ice to be 1.31, show that the Sundogs are located at an angle of 22 degrees from the Sun.

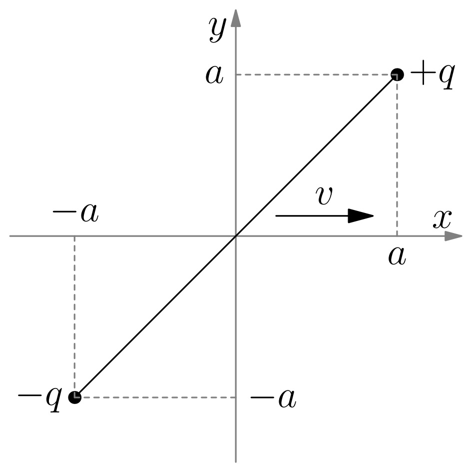
## Question 8

A positive electric charge $Q$ is located halfway between a pair of parallel infinite, grounded metal plates which are separated by a distance $d$.

1. Sketch a 2-D cross-section of the configuration and the electric field lines.
2. Determine the charge induced on each plate.
3. Determine the force acting on each plate.

## Question 9

The following is an idealization of the Trouton-Noble experiment.
Two small particles of charges $q = 1\mu C$ and $-q = -1\mu C$ are fixed at the ends of a thin non-conducting rod. For simplicity, assume that the rod is sufficiently thin not to perturb the electric or magnetic field produced by the charges. The assembly is not rotating and, in the lab (inertial) frame, it has been traveling at constant velocity $\vec{v} = (10^6\ \text{m/s})\hat{x}$ for a very long time prior to $t = 0$. At time $t = 0$, the charges' positions are respectively $(a, a)$ and $(-a, -a)$ in the $x$, $y$ plane, as illustrated.




Assuming $a = 4$ cm:

1. Calculate the total electric and magnetic forces exerted by each charge on the other charge in the lab frame.
2. Calculate the torque $\vec{N}$ associated with the sum of the electric and magnetic forces on each charge separately and on the whole assembly in the lab frame.
3. In the charges' rest frame there would be no magnetic field, the only forces at play would be electrostatic and they would not exert any torque on the charges. If left to itself, the assembly would therefore not start spinning. Yet, in the lab frame, electromangetic forces exert a torque which, it seems, would induce a spin. Explain this apparent paradox.

**Question 10**

A beam of spin 1/2 particles is run through a non uniform magnetic field in order to separate spin up and spin down particles, as in the Stern Gerlach experiment. Here spin up is measured with respect to the $z$ axis of a right handed system of coordinates and the beam coming out of the Stern Gerlach device propagates in the $+x$ direction. Unfortunately, the apparatus is faulty and 10% of the particles in the beam which should have their spin up are actually unpolarized. This imperfect 'spin up' beam is then used in a second experiment to measure the average spin $s_u$ of particles along the unit vector $\hat{u} = (0, \sqrt{2}/2, \sqrt{2}/2)$.

1. Determine the expectation value of $s_u$ in that second experiment.
2. If 10,000 particles are used in the second experiment, calculate the uncertainty (error bar) that should be placed on a 10% estimate of the fraction of unpolarized beam.